

**Z BOSON OBSERVABLES IN THE MSSM\***A. DABELSTEIN<sup>†</sup>, W. HOLLIK, W. MÖSLE<sup>†</sup>*Institut für Theoretische Physik, Universität Karlsruhe, Kaiserstr. 12  
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E-mail: add@dmumpiwh.bitnet***ABSTRACT**

A combined analysis for  $Z$  boson observables and the quantity  $\Delta r$  within the framework of the Minimal Supersymmetric Standard Model is presented. Contributions from supersymmetric particles are discussed and upper and lower bounds for the MSSM predictions are given.

**1. Introduction**

Precision data at LEP have reached a level of accuracy where the electroweak theory can be tested at its quantum structure. The predictions of the Standard Model for the precision observables at the  $Z$  resonance have been calculated with full one-loop and leading higher order contributions<sup>1,2</sup>. With two exceptions, all precision observables, measured at the level of a few per mille or even better, agree with the Standard Model predictions within  $1\sigma$ <sup>3</sup>. The top quark mass  $m_t = 180 \pm 12$  GeV as the weighted average of the CDF and DØ measurements<sup>4</sup> is in good agreement with the Standard Model implication from the precision data. The only mismatch is found in the ratios of the hadronic partial decay widths  $Z \rightarrow c\bar{c} (b\bar{b})$  to the total hadronic width :

$$R_c = \frac{\Gamma_c}{\Gamma_{had}}, \quad R_b = \frac{\Gamma_b}{\Gamma_{had}},$$

which deviate by  $\approx 2\sigma$  from the recent LEP data<sup>3</sup>.

It is not unlikely that the Standard Model around the  $Z$  scale is the effective low energy limit of a more comprehensive theory of the GUT class. Within these ideas the extension of the Standard Model to the Minimal Supersymmetric Standard Model (MSSM)<sup>5</sup> is of particular theoretical interest, allowing the unification of gauge couplings at the GUT scale  $\mathcal{O}(10^{15} \text{ GeV})$ <sup>6</sup>.

At present the MSSM is the most predictive framework for physics beyond the Standard Model. The superpartners appear in the electroweak radiative corrections

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and thus enter the theoretical predictions for the precision observables with the property that they decouple in the heavy mass limit. Precision measurements therefore provide a tool for indirect tests of the MSSM and for probing the possible mass range of new particles beyond the present limits from negative direct searches.

As an extension of the Standard Model, the general 2-Higgs doublet has been discussed for  $Z$  boson decays<sup>7</sup> and implications from electroweak precision data are presented in ref.<sup>8</sup>, also for a supersymmetric Higgs sector under the assumption of decoupling genuine SUSY particles. Recently, supersymmetric contributions for individual  $Z$  boson decay widths and ratios  $R_b$ ,  $R_c$  were presented in refs<sup>9,10,11,12,13</sup>.

In this article we describe the results of a complete MSSM 1-loop calculation for a combined discussion of all  $Z$  observables and  $M_W$ . The  $Z$  boson observables together with the quantity  $\Delta r$  (or  $M_W$ ) depend sizeably on the masses and parameters of the virtual standard and supersymmetric particles. For each observable the effects from the parameters in the various relevant MSSM sectors are discussed and comparisons with the minimal Standard Model are performed. Considering the  $Z$  boson observables and  $\Delta r$  globally as functions of the MSSM parameter set, the following conclusions can be observed:

- the MSSM can improve the agreement of the individually predicted observables with the experimental data. It is, however, difficult to remove the discrepancies from all observables simultaneously.
- a  $\chi^2$  fit for all input parameters gives a preferred set of MSSM parameters. This numerical analysis yields constraints on the supersymmetric particle masses and parameters. A prediction for a supersymmetric particle spectrum can not be achieved without additional assumptions, since the precision observables are only weakly sensitive to equal supersymmetric masses  $\geq 500$  GeV<sup>14</sup>.

In the following, the considered  $Z$  boson observables and their notation are introduced. A brief description of the one-loop and higher order corrections of the precision observables is presented and the parametrization of the MSSM is given. The discussion compares the MSSM predictions with the experimental data and presents constraints on the supersymmetric parameter space.

## 2. $Z^0$ boson on-resonance observables

At the  $Z$  boson resonance two classes of precision observables are available:

a) inclusive quantities:

- the partial leptonic and hadronic decay width  $\Gamma_{f\bar{f}}$ ,
- the total decay width  $\Gamma_Z$ ,

- the hadronic peak cross section  $\sigma_h$ ,
- the ratio of the hadronic to the electronic decay width of the  $Z$  boson:  $R_h$ ,
- the ratio of the partial decay width for  $Z \rightarrow c\bar{c} (b\bar{b})$  to the hadronic width,  $R_c, R_b$ .

b) asymmetries and the corresponding mixing angles:

- the *forward-backward* asymmetries  $A_{FB}^f$ ,
- the *left-right* asymmetries  $A_{LR}^f$ ,
- the  $\tau$  polarization  $P_\tau$ ,
- the effective weak mixing angles  $\sin^2 \theta_{eff}^f$ .

Together with the quantity  $\Delta r$  in the correlation of the  $W$  mass to the electroweak input parameters  $G_\mu, M_Z$  and  $\alpha_{EM}$ , this set of precision observables is convenient for a numerical analysis of the supersymmetric parameter space.

### 2.1. The effective $Z$ - $f$ - $f$ couplings

The coupling of the  $Z$  boson to fermions  $f$  can be expressed by effective vector and axial vector coupling constants  $v_{eff}^f, a_{eff}^f$  in terms of the NC vertex:

$$J_{NC}^\mu = \frac{e}{2s_W^2 c_W^2} \gamma^\mu (v_{eff}^f - a_{eff}^f \gamma_5) , \quad (1)$$

where the convention is introduced :  $c_W^2 = \cos^2 \theta_W = 1 - s_W^2 = M_W^2/M_Z^2$ <sup>15</sup>. Input parameters are the  $\mu$  decay constant  $G_\mu = 1.166392 \times 10^{-5} \text{ GeV}^{-2}$ ,  $\alpha_{EM} = 1/137.036$  and the mass of the  $Z^0$  boson  $M_Z = 91.1887 \text{ GeV}$ . The mass of the  $W$  boson is related to these input parameters through:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi \alpha_{EM}}{2s_W^2 M_W^2} \cdot \frac{1}{1 - \Delta r_{MSSM}(\alpha_{EM}, M_W, M_Z, m_t, \dots)} , \quad (2)$$

where the complete MSSM one-loop contributions are parametrized by the quantity  $\Delta r_{MSSM}$ <sup>16</sup>. Leading higher order Standard Model corrections<sup>1,17</sup> to the quantity  $\Delta r$  are included in the calculation.

The effective couplings  $v_{eff}^f, a_{eff}^f$  can be written as:

$$\begin{aligned} v_{eff}^f &= \sqrt{Z_Z} (v^f + \Delta v^f + Z_M Q_f) \\ a_{eff}^f &= \sqrt{Z_Z} (a^f + \Delta a^f) . \end{aligned} \quad (3)$$

$v^f$  and  $a^f$  are the tree-level vector and axial vector couplings:

$$v^f = I_3^f - 2Q_f s_W^2 , \quad a^f = I_3^f . \quad (4)$$

$Z_Z$ ,  $Z_M$  are given in Eq. (9). The complete MSSM one-loop contributions of the non-universal finite vector and axial vector couplings  $\Delta v^f$ ,  $\Delta a^f$  have been calculated<sup>14</sup>, together with the leading two-loop Standard Model contributions<sup>1,2</sup>. They are derived in the 't Hooft-Feynman gauge and in the on-shell renormalization scheme<sup>18</sup>.

The universal finite  $Z$  boson wave function renormalization  $Z_Z$  and the  $\gamma Z$  mixing  $Z_M$  are derived from the  $(\gamma, Z)$  propagator matrix. The inverse matrix is:

$$(\Delta_{\mu\nu})^{-1} = ig_{\mu\nu} \begin{pmatrix} k^2 + \hat{\Sigma}_\gamma(k^2) & \hat{\Sigma}_{\gamma Z}(k^2) \\ \hat{\Sigma}_{\gamma Z}(k^2) & k^2 - M_Z^2 + \hat{\Sigma}_Z(k^2) \end{pmatrix}, \quad (5)$$

where  $\hat{\Sigma}_\gamma$ ,  $\hat{\Sigma}_Z$ ,  $\hat{\Sigma}_{\gamma Z}$  are the renormalized self energies and mixing. They are obtained by summing the loop diagrams and the counter terms<sup>1</sup>.

The entries in the  $(\gamma, Z)$  propagator matrix:

$$\Delta_{\mu\nu} = -ig_{\mu\nu} \begin{pmatrix} \Delta_\gamma & \Delta_{\gamma Z} \\ \Delta_{\gamma Z} & \Delta_Z \end{pmatrix}, \quad (6)$$

are given by:

$$\begin{aligned} \Delta_\gamma(k^2) &= \frac{1}{k^2 + \hat{\Sigma}_\gamma(k^2) - \frac{\hat{\Sigma}_{\gamma Z}^2(k^2)}{k^2 - M_Z^2 + \hat{\Sigma}_Z(k^2)}} \\ \Delta_Z(k^2) &= \frac{1}{k^2 - M_Z^2 + \hat{\Sigma}_Z(k^2) - \frac{\hat{\Sigma}_{\gamma Z}^2(k^2)}{k^2 + \hat{\Sigma}_\gamma(k^2)}} \\ \Delta_{\gamma Z}(k^2) &= -\frac{\hat{\Sigma}_{\gamma Z}(k^2)}{[k^2 + \hat{\Sigma}_\gamma(k^2)][k^2 - M_Z^2 + \hat{\Sigma}_Z(k^2)] - \hat{\Sigma}_{\gamma Z}^2(k^2)}. \end{aligned} \quad (7)$$

The renormalization condition to define the mass of the  $Z$  boson is given by the pole of the propagator matrix Eq. (5). The pole  $k^2 = M_Z^2$  is the solution of the equation:

$$\mathcal{R}e[(M_Z^2 + \hat{\Sigma}_\gamma(M_Z^2))\hat{\Sigma}_Z(M_Z^2) - \hat{\Sigma}_{\gamma Z}^2(M_Z^2)] = 0. \quad (8)$$

Eq. (7) yields the wave function renormalization  $Z_Z$  and mixing  $Z_M$ :

$$\begin{aligned} Z_Z &= \text{Res}_{M_Z} \Delta_Z = \frac{1}{1 + \hat{\Sigma}'_Z(k^2) - \left( \frac{\hat{\Sigma}_{\gamma Z}^2(k^2)}{k^2 + \hat{\Sigma}_\gamma(k^2)} \right)} \Bigg|_{k^2=M_Z^2} \\ Z_M &= -\frac{\hat{\Sigma}_{\gamma Z}(M_Z^2)}{M_Z^2 + \hat{\Sigma}_\gamma(M_Z^2)}. \end{aligned} \quad (9)$$

## 2.2. $Z$ boson observables

The fermionic  $Z$  boson partial decay widths  $\Gamma_{f\bar{f}}$  can be written:

1)  $f \neq b$ :

$$\Gamma_{f\bar{f}} = \frac{N_C \alpha_{EM} M_Z}{3} \sqrt{1 - \frac{4m_f^2}{M_Z^2}} \left[ (v_{eff}^f)^2 \left(1 + \frac{2m_f^2}{M_Z^2}\right) + (a_{eff}^f)^2 \left(1 - \frac{4m_f^2}{M_Z^2}\right) \right] \cdot \left(1 + \frac{3\alpha_{EM}}{4\pi} Q_f^2\right) (1 + \delta_{QCD}^f), \quad (10)$$

where

$$\delta_{QCD}^f = \begin{cases} 0 & , f = \text{leptons} \\ \frac{\alpha_s}{\pi} + 1.405 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.8 \left(\frac{\alpha_s}{\pi}\right)^3 & , f = \text{quarks} \end{cases}. \quad (11)$$

2)  $f = b$ :

$$\Gamma_{b\bar{b}} = \alpha_{EM} M_Z \left[ (v_{eff}^b)^2 + (a_{eff}^b)^2 \right] \cdot \left(1 + \frac{3\alpha_{EM}}{4\pi} Q_b^2\right) (1 + \delta_{QCD}^b) + \Delta\Gamma_{b\bar{b}}. \quad (12)$$

In  $\Delta\Gamma_{b\bar{b}}$  the  $b$  quark specific finite mass terms and QCD corrections together with the leading two-loop  $\mathcal{O}(\alpha_{EM}\alpha_S)$  are included<sup>1,2</sup>.  $\delta_{QCD}^b$  is given in Eq. (11).

The total decay width  $\Gamma_Z$  is the sum of the leptons and quarks:

$$\Gamma_Z = \sum_f \Gamma_{f\bar{f}}. \quad (13)$$

In the following  $\Gamma_{had} = \sum_q \Gamma_{q\bar{q}}$  is the hadronic decay width of the  $Z$  boson.

The hadronic peak cross section:

$$\sigma_h = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_Z^2}. \quad (14)$$

The ratio of the hadronic to the electronic decay width is defined:

$$R_e = \frac{\Gamma_{had}}{\Gamma_{ee}}. \quad (15)$$

The ratio of the partial decay width for  $Z \rightarrow b\bar{b} (c\bar{c})$  to the total hadronic decay width:

$$R_{b(c)} = \frac{\Gamma_{b\bar{b}(c\bar{c})}}{\Gamma_{had}}. \quad (16)$$

The following quantities and observables depend on the ratio of the vector to the axial vector coupling. The effective flavour dependent weak mixing angle can be written:

$$\sin^2 \theta_{eff}^f = \frac{1}{4|Q_f|} \left( 1 - \frac{v_{eff}^f}{a_{eff}^f} \right). \quad (17)$$

Table 1: Experimental LEP and  $p\bar{p}$  results.

$\Gamma_Z$ [GeV]	2.4971	$\pm$	0.0032
$\sigma_h$ [GeV]	41.492	$\pm$	0.081
$R_e$	20.843	$\pm$	0.060
$R_\mu$	20.805	$\pm$	0.048
$R_\tau$	20.798	$\pm$	0.066
$A_{FB}^e$	0.0154	$\pm$	0.0030
$A_{FB}^\mu$	0.0160	$\pm$	0.0017
$A_{FB}^\tau$	0.0209	$\pm$	0.0024
$R_b$	0.2204	$\pm$	0.0020
$R_c$	0.1606	$\pm$	0.0095
$A_{FB}^b$	0.1015	$\pm$	0.0036
$A_{FB}^c$	0.0760	$\pm$	0.0089
$\Delta r$	0.0425	$\pm$	0.009

The *left-right* asymmetries:

$$A_{LR}^f = \mathcal{A}^f = \frac{2 v_{eff}^f / a_{eff}^f}{1 + (v_{eff}^f / a_{eff}^f)^2} . \quad (18)$$

The *forward-backward* asymmetries:

$$A_{FB}^f = \frac{3}{4} \mathcal{A}^e \mathcal{A}^f . \quad (19)$$

### 3. Discussion

The experimental results from the recent LEP and  $p\bar{p}$  data are summarized in table 1<sup>3,19</sup>. In addition to the measurements, the correlation matrix<sup>3</sup> is used. The value for  $\alpha_S(M_Z^2)$  has been fixed from QCD observables at the Z pole<sup>20</sup>:

$$\alpha_S(M_Z^2) = 0.123 \pm 0.006 , \quad (20)$$

and the  $b$  quark mass  $m_b = 4.7$  GeV. In the following we discuss the general version of the MSSM without further restrictions to the parameters from assumptions about Grand Unification or Supergravity.

#### 3.1. Higgs sector

The Higgs sector of the MSSM is that of a two Higgs doublet model, where the coefficients of the potential are restricted by supersymmetry. As a consequence of the

supersymmetric Higgs potential, a light Higgs boson exists with a tree level upper mass bound given by the  $Z$  boson mass. The Higgs potential contains two independent free parameters, which are  $\tan\beta = v_2/v_1$  and the mass of the pseudoscalar Higgs boson  $M_{A^0}$ .  $v_1, v_2$  are the vacuum expectation values of the Higgs doublets.

Radiative corrections to the Higgs mass spectrum predict an upper limit of the light Higgs mass  $\mathcal{O}(130 \text{ GeV})^{21}$ . In the calculation of the neutral scalar MSSM Higgs mass spectrum and the mixing angle  $\alpha$  the corrections  $\mathcal{O}(m_t^4)$  are included<sup>22</sup>.

Fig. 1 shows the dependence of a subset of observables:  $\Delta r$ ,  $R_b$ ,  $R_c$ ,  $R_h = R_e$ ,  $\Gamma_{tot}$ ,  $\Gamma_{ee}$ ,  $\sin^2\theta_{eff}^e$ ,  $A_{FB}^b$  and  $A_{FB}^c$  as functions on  $M_A$  for values  $\tan\beta = 0.7$  (dotted), 1.5 (long dotted), 8 (dashed-dotted), 20 (dashed), 70 (solid). This set of observables is convenient for a qualitative discussion of the universal and non-universal MSSM one-loop contributions. The top quark mass is fixed at  $m_t = 174 \text{ GeV}$ , and squark, slepton masses are  $m_{\tilde{q}} = 500 \text{ GeV}$ ,  $m_{\tilde{l}} = 800 \text{ GeV}$  without left-right mixing. The soft breaking parameters in the gaugino sector are  $\mu = 100 \text{ GeV}$ ,  $M = 300 \text{ GeV}$  and a heavy gluino mass  $m_{\tilde{g}} = 800 \text{ GeV}$  is fixed. This set of genuine supersymmetric masses does not show sizeable effects on these observables and allows to explore only the SUSY Higgs sector.

A complete discussion on the quantity  $\Delta r$  ( $\delta M_W$ ) is available<sup>16</sup>, however, the inclusion of  $\mathcal{O}(m_t^4)$  Higgs mass and  $\mathcal{O}(\alpha_{EM}\alpha_S^2)$  contributions<sup>22,1</sup> improve the theoretical  $\Delta r$  predictions. In Fig. 1 the quantity  $\Delta r$  increases with  $M_A$  and is almost constant above  $M_A \geq 250 \text{ GeV}$ . Small  $\tan\beta = 0.7$  values give sizeable positive contributions to  $\Delta r$ , since radiative corrections to the scalar Higgs mass are large in the regime of small  $M_A < 100 \text{ GeV}$  and small  $\tan\beta$ . In the mass range  $M_A < 70 \text{ GeV}$  and  $\tan\beta \geq 2$  the quantity  $\Delta r$  decreases and disfavors with the experimental data within the range of the measured top quark mass. Contributions of genuine supersymmetric masses (next subsection) do not increase  $\Delta r$  and enhance the deviation from the experimental result even stronger. A similar conclusion for the parameters  $M_A$ ,  $\tan\beta$  is obtained in Fig. 1 for the quantity  $\sin^2\theta_{eff}^e$ .

In the  $\tan\beta < 1$  range the ratio  $R_b$  decreases for values  $M_A < 500 \text{ GeV}$ . This effect is strong for small  $M_A \approx 50 \text{ GeV}$  and yields the Standard Model results for  $M_A \rightarrow \infty$ . For intermediate  $\tan\beta$ ,  $4 < \tan\beta < 30$ ,  $R_b$  is nearly constant for  $50 \text{ GeV} \leq M_A \leq 500 \text{ GeV}$  and agrees with the Standard Model prediction. Large  $\tan\beta \approx 70$  enhance  $R_b$  for small  $M_A \leq 55 \text{ GeV}$ , within the experimental  $1\sigma$  bounds. In Fig. 1 this peak at  $M_A = 46 \text{ GeV}$  is shown, however this scenario would predict a rather light neutral Higgs mass  $M_{h^0}$ . For  $M_A \geq 75 \text{ GeV}$  the ratio  $R_b$  decreases, reaches a minimum for  $M_A = 150 \text{ GeV}$  and approaches the SM result for larger pseudoscalar masses. The ratio  $R_c$  in Fig. 1 lies above the experimental  $1\sigma$  bounds. Recently, the complementary  $R_b - R_c$  effects have also been discussed<sup>11</sup>. However, the  $R_c$  sensitivity on the Higgs sector is small and only for large  $\tan\beta = 70$  and  $40 \text{ GeV} < M_A < 60 \text{ GeV}$  the agreement with the experimental data is improved.

The total decay width  $\Gamma_{tot}$  and the ratio  $R_h$  in Fig. 1 show a qualitatively similar line shape as the ratio  $R_b$ .  $\Gamma_{tot}$  and  $R_h$  depend strongly on  $\alpha_s$ , however, the “peak” region in  $R_b$  for  $M_A < 70$  GeV and small (large)  $\tan\beta < 1(> 40)$  yields values for the quantities  $\Gamma_{tot}$ ,  $R_h$  which are larger than the experimental results. A better agreement can be obtained by decreasing the total value of  $\alpha_s$ .

### 3.2. Sfermion and Gaugino/Higgsino sector

Sfermions and charginos (neutralinos) appear in the vertex correction to  $Z$  boson decays and give sizeable contributions for the external  $b$  state. The chargino (neutralino) masses and the mixing angles in the gaugino couplings are calculated from soft breaking parameters  $M$ ,  $M'$  and  $\mu$  in the chargino (neutralino) mass matrix<sup>5</sup>. The validity of the GUT relation  $M' = 5/3 \tan\theta_W M$  is assumed.

Squarks and sleptons are described by a  $2 \times 2$  mass matrix:

$$\mathcal{M}_{\tilde{f}}^2 = \begin{pmatrix} M_{\tilde{Q}}^2 + m_f^2 + M_Z^2(I_3^f - Q_f s_W^2) \cos 2\beta & m_f(A_f + \mu\{\cot\beta, \tan\beta\}) \\ m_f(A_f + \mu\{\cot\beta, \tan\beta\}) & M_{\{\tilde{U}, \tilde{D}\}}^2 + m_f^2 + M_Z^2 Q_f s_W^2 \cos 2\beta \end{pmatrix}, \quad (21)$$

with SUSY soft breaking parameters  $M_{\tilde{Q}}$ ,  $M_{\tilde{U}}$ ,  $M_{\tilde{D}}$ ,  $A_f$ , and  $\mu$ . The notation in the off-diagonal entries in Eq. (21):

$$A'_f = A_f + \mu\{\cot\beta, \tan\beta\}, \quad (22)$$

will be used. Up and down type sfermions in (21) are distinguished by setting  $f=u,d$  and the  $\{u, d\}$  entries in the parenthesis. Instead of  $M_{\tilde{Q}}$ ,  $M_{\tilde{U}}$ ,  $M_{\tilde{D}}$ ,  $A'_b$ ,  $A'_t$ , the physical squark masses are given by  $m_{\tilde{b}} = m_{\tilde{b}_L} = m_{\tilde{b}_R}$ ,  $m_{\tilde{t}_2}$  and  $A'_t$ . No left-right mixing for  $\tilde{b}$  squarks is assumed and the  $\tilde{u}$ ,  $\tilde{d}$ ,  $\tilde{c}$ ,  $\tilde{s}$  masses are equal to the  $\tilde{b}$  squark mass. The sneutrino masses  $m_{\tilde{\nu}}$  are given without left-right mixing for sleptons.

In Fig. 2 the dependence of the  $Z$  boson observables and  $\Delta r$  for sbottom masses  $m_{\tilde{b}} = 150$  (500) GeV, and stop masses  $m_{\tilde{t}_R} = 50$  GeV (solid line), 150 GeV (dotted, dotted-dashed), 300 (500) GeV (long dotted, dashed) is shown as a function of  $\tan\beta$ . No left-right mixing in the stop states is assumed.  $m_t = 174$  GeV,  $\mu = 100$  GeV,  $M = 100$  GeV,  $M_A = 800$  GeV,  $m_{\tilde{l}} = 900$  GeV and  $m_{\tilde{g}_L} = 900$  GeV.

Sfermion and chargino (neutralino) contributions decrease the quantity  $\Delta r$ <sup>16</sup>. Charginos and neutralinos give numerically small effects, large  $\tilde{u}_L$ - $\tilde{d}_L$  mass splittings in the squark and slepton sector are the dominant supersymmetric contributions. The quantity  $\Delta r$  gets sizeable contributions from the  $\rho$  parameter:

$$\Delta r \simeq \Delta\alpha_{EM} - \frac{c_W^2}{s_W^2} \Delta\rho + \dots. \quad (23)$$



The  $\tilde{b}_L$ - $\tilde{t}_L$  mass splitting is described by the  $\Delta\rho$  parameter<sup>16</sup>:

$$\Delta\rho_{\tilde{b}-\tilde{t}}^0 = \frac{3\alpha_{EM}}{16\pi s_W^2 M_W^2} \left( m_{\tilde{b}_L}^2 + m_{\tilde{t}_L}^2 - 2 \frac{m_{\tilde{b}_L}^2 m_{\tilde{t}_L}^2}{m_{\tilde{b}_L}^2 - m_{\tilde{t}_L}^2} \log \frac{m_{\tilde{b}_L}^2}{m_{\tilde{t}_L}^2} \right), \quad (24)$$

and decreases  $\Delta r$  for light  $\tilde{b}$  masses<sup>a</sup>. Fig. 2 shows  $\Delta r$  for sbottom masses  $m_{\tilde{b}} = 150$  (500) GeV, where a light  $\tilde{b}$  mass decreases  $\Delta r$  sizeably. A variation in the stop state masses  $m_{\tilde{t}_R}$  in Fig. 2 shows small contributions on  $\Delta r$ . The effect on left-right sfermion mixing is discussed in Fig. 3.

An enhancement of the ratio  $R_b$  can be observed for light charginos  $\mathcal{O}(100$  GeV) and light stop-sbottom masses. This effect is stronger for small (large)  $\tan\beta < 1$  ( $> 40$ ) values due to the  $\tan\beta$  enhanced Yukawa type chargino (neutralino) - quark - squark couplings. Taking the present experimental limits on the genuine supersymmetric and Higgs masses into account,  $R_b$  can not be increased within the experimental  $1\sigma$  bound. The enhancement in  $R_b$  is larger for light  $\tilde{t}_R$  masses in Fig. 2, whereas  $R_b$  is nearly insensitive to the sbottom mass and therefore the  $\tilde{t}_L$  mass.

The ratio  $R_h$  and the total decay width  $\Gamma_{tot}$  increase for light left or right sfermion states in Fig. 2. For  $m_t = 174$  GeV and  $\alpha_s = 0.123$ , a light  $\tilde{b}$  squark  $m_{\tilde{b}} = 150$  GeV increases  $\Gamma_{tot}$  by about  $2\sigma$  above the present experimental data. This enhancement for light sbottom masses is shown for the electronic decay width  $\Gamma_{ee}$ , where no sensitivity for  $\tilde{t}_R$  masses can be observed. The *forward-backward* asymmetry  $A_{FB}^b$  also favours larger sbottom masses and shows smaller effects from  $m_{\tilde{t}_R}$ . Light  $\tilde{b}_L$  masses increase the *forward-backward* asymmetries  $A_{FB}^b$ ,  $A_{FB}^c$ .

Sfermion mixing for the parameter set in Fig. 2 “screens” the contributions from supersymmetric particles for all displayed observables. In Fig. 3,  $m_{\tilde{b}} = 150$  GeV,  $m_{\tilde{t}_2} = 50$  GeV and all other parameters are as in Fig. 2. The stop mixing is shown for  $A'_t = -930$  GeV (dotted-dashed),  $-464$  GeV (dashed),  $0$  (solid),  $464$  GeV (dotted),  $930$  GeV (long dotted). These  $A'_t$  mixing values yield  $\tilde{t}_1$  masses  $m_{\tilde{t}_1} = 800, 442, 216, 442, 800$  GeV. No (very small) stop mixing decreases  $\Delta r$  maximal, increases the ratio  $R_b$  maximal, etc. As a result, sfermion mixing can screen the supersymmetric radiative corrections for precision observables, even if supersymmetric particles are light  $\mathcal{O}(100$  GeV).

Fig. 4 shows the  $Z$  boson observables and  $\Delta r$  for chargino (neutralino) masses as functions of  $\mu$  for fixed  $M = 100$  GeV. Stop masses are  $m_{\tilde{t}_R} = 50$  GeV (solid line),  $150$  GeV (dotted),  $800$  GeV (long dotted) and  $m_{\tilde{b}} = 500$  GeV without left-right mixing.  $m_t = 174$  GeV and large  $A^0$ , slepton and gluino masses are assumed:  $M_A = 800$  GeV,  $m_{\tilde{l}} = 900$  GeV,  $m_{\tilde{g}} = 800$  GeV. In Fig. 4  $\tan\beta = 1.1$  and the experimentally excluded chargino masses are indicated by the shadowed  $\mu$  range ( $m_{\tilde{\chi}^\pm} \geq 48$  GeV). Close to the experimentally excluded  $\mu$  region the contribution from charginos and sfermions is largest. The ratio  $R_b$  increases within the experimental  $1\sigma$  range for

<sup>a</sup>The superscript  $\Delta\rho^0$  indicates that no left-right mixing is present.

$m_{\tilde{\chi}^\pm} = 50$  GeV and  $m_{\tilde{t}_R} = 50$  GeV. Larger chargino/stop masses decouple above  $\mathcal{O}(500)$  GeV from the discussed observables. Larger  $\tan\beta$  values give a smaller but qualitative similar result for  $R_b$ . A small value  $\tan\beta = 1.1$  decreases  $\Delta r$  also for light  $\tilde{t}_R$  masses. The ratio  $R_h$ ,  $\sin^2\theta_{eff}$  and the *forward-backward* asymmetry  $A_{FB}^b$  are in better agreement with the experimental data for large chargino/stop masses or strong sfermion mixing.

### 3.3. Upper and lower bounds at LEP I and LEP II

Fig. 5 shows the overall upper and lower bounds of the theoretical predictions for each observable individually. The results are plotted as a function of the top quark mass  $130 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$  for the Standard Model (dashed) and the MSSM (solid). The standard Higgs mass is varied between  $66 \text{ GeV} \leq M_{H_{SM}} \leq 800 \text{ GeV}$ . The MSSM parameter set is restricted by the present mass bounds from direct searches at *LEP I* and  $p\bar{p}$  data as shown by the solid MSSM lines in Fig. 5. Mass bounds from direct sparticle searches to be expected at *LEP II* are shown by dotted lines. An overlap between the SM and MSSM upper and lower bounds is given for large decoupled genuine supersymmetric particles and a light neutral scalar MSSM Higgs mass:  $M_{h_{upper}^0} > M_{H_{SM},lower}$ . The experimental results are indicated by the dark bounds.

In case that no direct signal for supersymmetry is seen at *LEP II*, upper and lower MSSM bounds on  $R_b$  ( $R_c$ ) can not explain the presently observed discrepancy with the experimental data. In conclusion from the previous results and Fig. 5 the MSSM can not improve the predictions for precision observables for light sfermions  $\leq 300$  GeV. Constraints on sfermion masses for top quark masses  $160 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$  and  $\alpha_s = 0.118\dots 0.128$  prefer squarks  $m_{\tilde{q}} > 300$  GeV, if no squark mixing is assumed. Large sfermion mixings, however, can not exclude a light stop quark  $\mathcal{O}(100 \text{ GeV})$ .

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## 6. Figure Captions

**Figure 1.** The observables  $\Delta r$ ,  $R_b$ ,  $R_c$ ,  $\Gamma_{tot}$ ,  $\Gamma_{ee}$ ,  $\sin^2 \theta_{eff}^e$ ,  $A_{FB}^b$ ,  $A_{FB}^c$  are plotted as a function on the pseudoscalar mass  $M_A$  for values  $\tan \beta = 0.7$  (dotted line), 1.5 (long dotted), 8 (dashed-dotted), 20 (dashed) and 70 (solid).  $m_t = 174$  GeV, squark and slepton masses are  $m_{\tilde{q}} = 500$  GeV,  $m_{\tilde{l}} = 800$  GeV, no left-right mixing.  $\mu = 100$  GeV,  $M = 300$  GeV and the gluino mass  $m_{\tilde{g}} = 800$  GeV.

**Figure 2.** The set of observables from Fig. 1. as a function on  $\tan \beta$  for squark masses  $m_{\tilde{b}} = 150$  (500) GeV and  $m_{\tilde{t}_R} = 50$  GeV (solid line), 150 GeV (dotted), 300 (500) GeV (long dotted, dashed), no left-right mixing.  $m_t = 174$  GeV,  $\mu = 100$  GeV,  $M = 100$  GeV,  $M_A = 800$  GeV,  $m_{\tilde{l}} = 900$  GeV and  $m_{\tilde{g}} = 900$  GeV.

**Figure 3.** The set of observables from Fig. 1. as a function on  $\tan \beta$  for stop mixing values  $A_{\tilde{t}} = -930$  GeV (dotted-dashed),  $-464$  GeV (dashed), 0 (solid), 464 GeV (dotted), 930 GeV (long dotted).  $m_{\tilde{b}} = 150$  GeV,  $m_{\tilde{t}_2} = 50$  GeV and all other parameters from Fig. 2.

**Figure 4.** The set of observables from Fig. 1. as a function on  $\mu$  for fixed  $M = 100$  GeV. Stop masses are  $m_{\tilde{t}_R} = 50$  GeV (solid line), 150 GeV (dotted) and 800 GeV (long dotted).  $m_{\tilde{b}} = 500$  GeV, no left-right mixing,  $m_t = 174$  GeV,  $M_A = 800$  GeV,  $m_{\tilde{l}} = 900$  GeV,  $m_{\tilde{g}} = 800$  GeV.

**Figure 5.** Upper and lower bounds of the individual observables as a function on  $m_t$  for the Standard Model (dashed line) and the MSSM (solid line). The MSSM parameters are restricted by the present mass bounds from direct searches at *LEPI* and *p $\bar{p}$*  colliders. Mass bounds from direct sparticle searches to be expected at *LEPII* are shown by the dotted lines.

Fig. 1

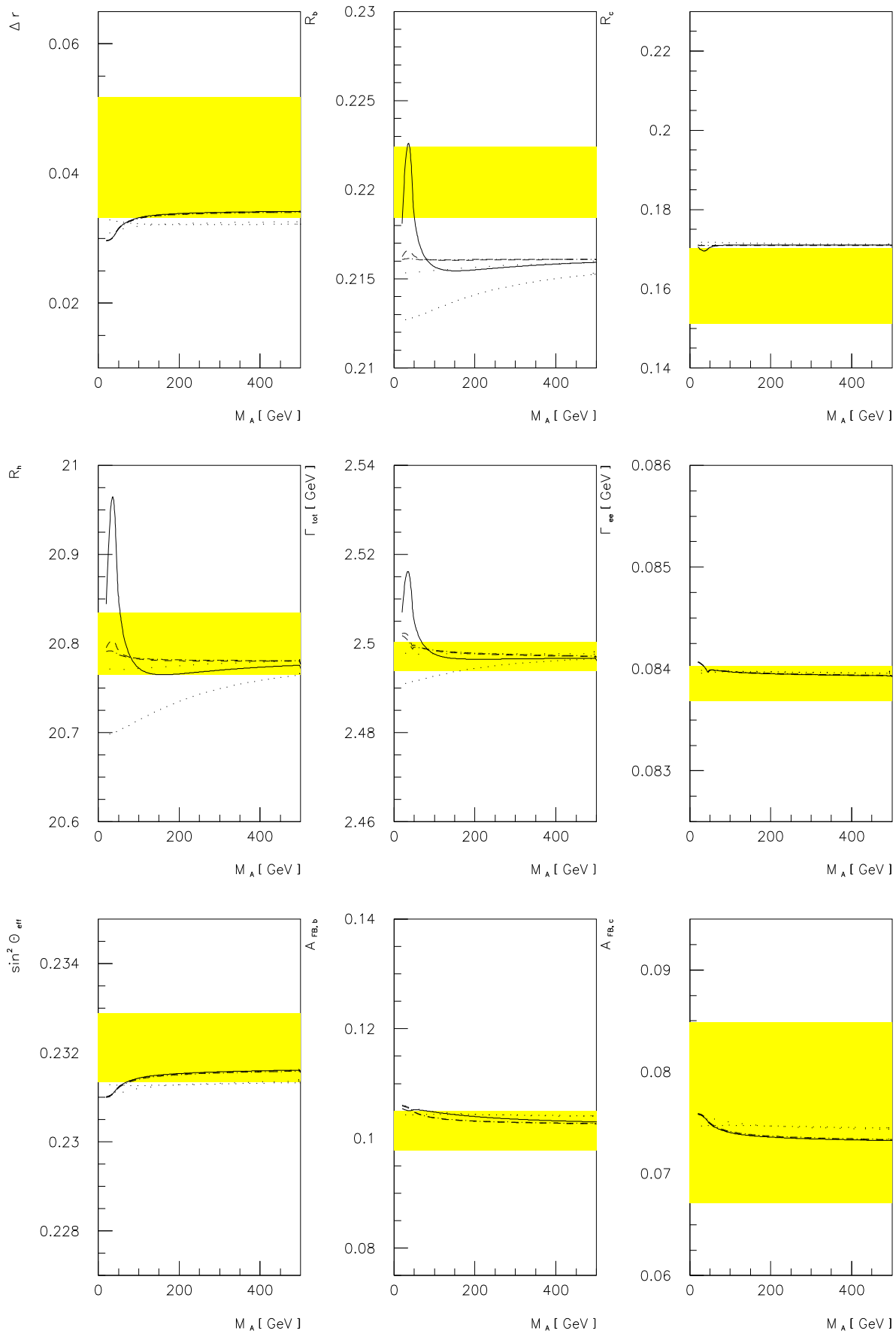


Fig. 2

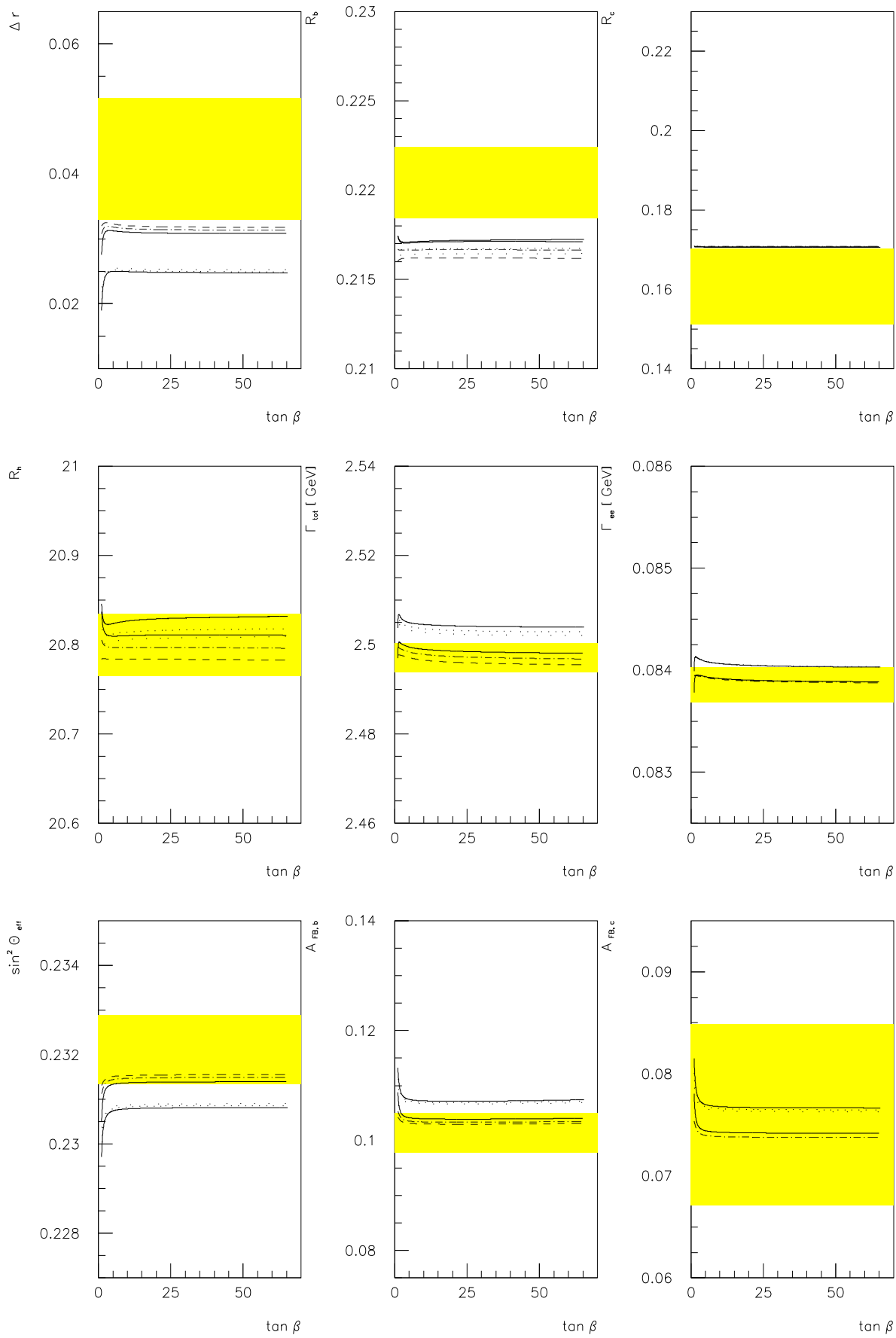


Fig. 3

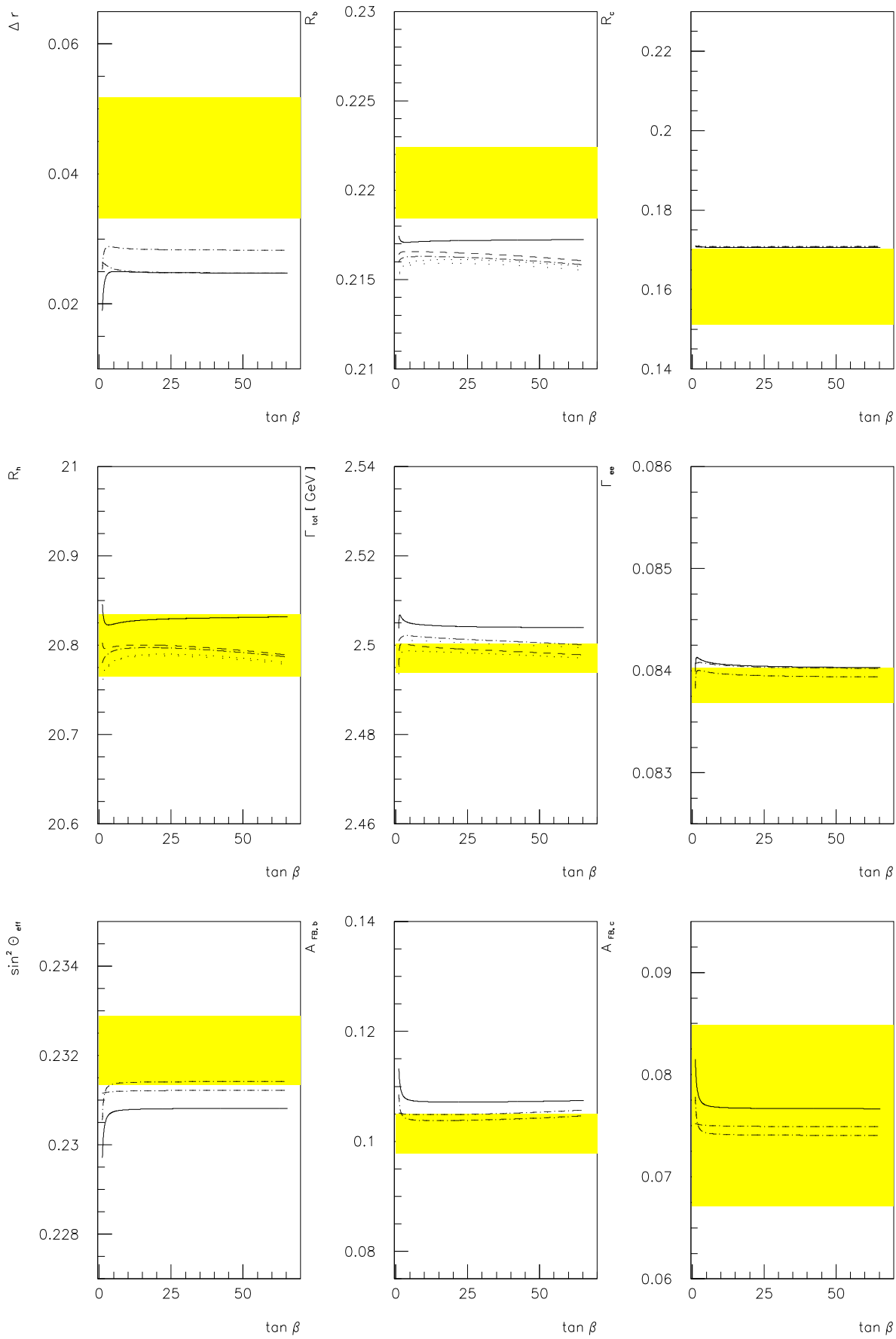




Fig. 4

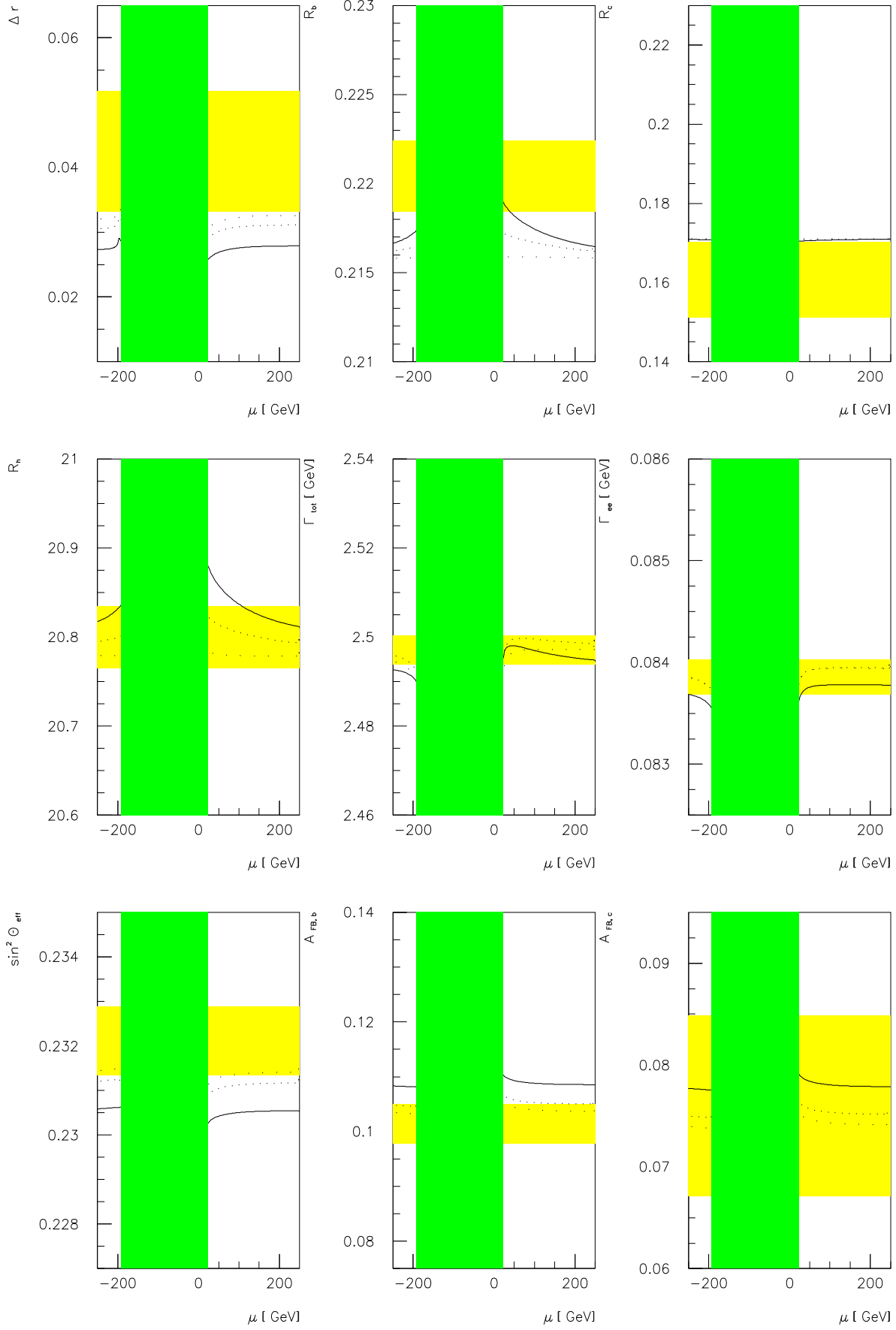


Fig. 5

